

Introduction To Algorithmic Trading

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Preliminary information

- **References** J. Hasbrouck's book "Empirical Market Microstructure" and papers
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- **Outline** Pricing, Risk Management, Execution, Automation

 $\gamma \text{ L A } \pi \equiv \nu \text{ C } \gamma$

Why Algorithmic Trading?

- Electronic trading is a common feature of modern financial markets
- Availability of computing power and large data sets
- Highly competitive environment, increased productivity



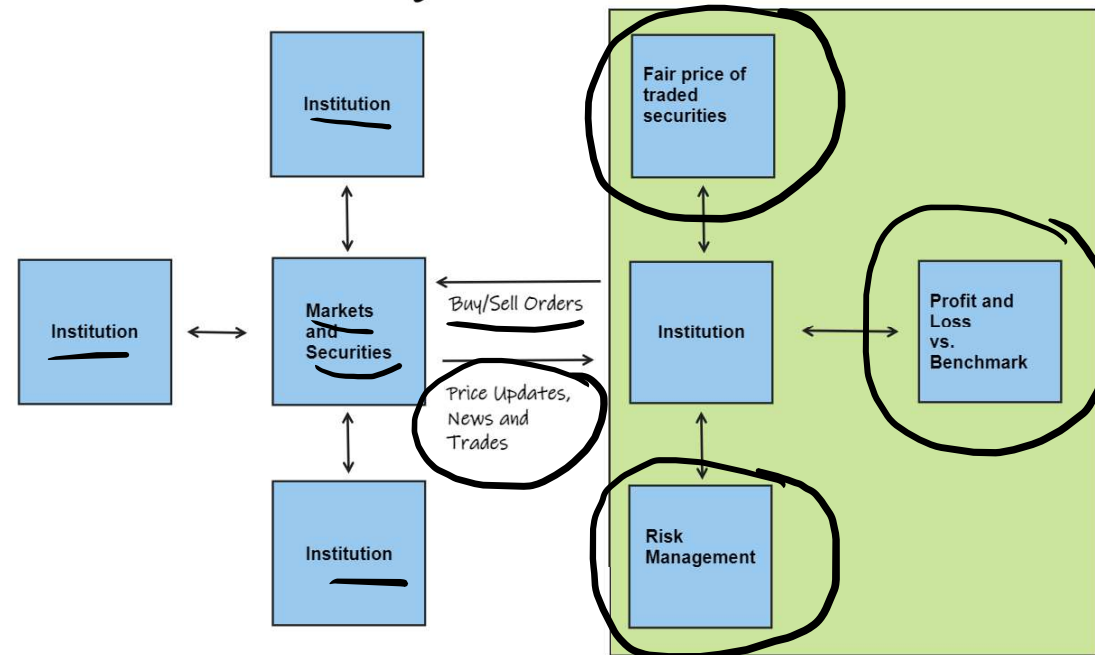
Syllabus

- **Introduction:** securities, markets, institutions and agents. Short term mean reversion: the Roll model of traded prices. Market microstructure and the Central Order Book.
- **The dealer problem:** market making and inventory control. An example of market making strategy.
- **Pricing and statistical arbitrage:** modeling securities using market invariants, Ornstein-Uhlenbeck processes, and Principal Component Analysis. Application to multivariate time series. Z-scores and drawbacks.
- **Optimal execution in a Limit Order Market:** Implementation shortfall. Limit orders vs. market orders. Order management systems and protocols. Event-driven programming. Filters. Regulation. Network latency.

Introduction/Terminology

- **Securities:** Fixed Income Bonds (Government/Supranational/Corporate), Currencies (FX/Crypto), Equities, Index and Single Stock Derivatives (Futures and Options)
- **Institutions:** Dealers/Market Makers/Liquidity Providers, Brokers, Asset Managers, Execution Desks, Hedge Funds, Individuals, Central Banks.
- **Market Models:** Electronic, Broker/Dealer, Lit/Dark, Continuous Trading vs Auctions, Bilateral vs Multilateral.

A trading environment.



Algorithmic Trading: an approach to trading using a predefined set of rules.



The quest for algorithmic trading strategies: structural vs data driven approach

Structural: first of all build a market model for your securities, then choose appropriate actions based on some performance criteria.

Purely Data Driven: optimize your performance function directly, using available data.

An example of a momentum trading strategy (highly simplified pseudo-code)

$$\{P_t\}_{t=1}^T \quad \hat{P}_t = \hat{P}_{t-1} \cdot \lambda + P_t \cdot (1 - \lambda)$$

$\uparrow$ EWMA $\lambda = 0.94$

Warning : just an example!

$$\begin{cases} P_t > \hat{P}_t \rightarrow \text{hold} \\ P_t < \hat{P}_t \rightarrow \text{short position} \end{cases}$$

A theoretical framework for a data driven approach to algorithmic trading

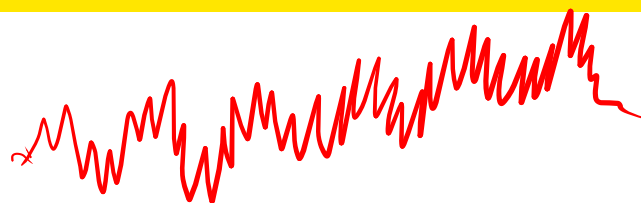
An algorithmic trading strategy could be modeled as a mapping from a **State Space** $\mathcal{S}$ to a set of **Actions** $\mathcal{A}$, using **Policy** π , where the probability of choosing action $a \in \mathcal{A}$ given state $s \in \mathcal{S}$ is $\pi(a|s)$.

For the simple momentum strategy discussed previously, the state at time t is given by $s_t = \{S.p, S.mmp\}$

$$\pi(a_t|s_t) : a_t = \begin{cases} \text{take long position} & \text{if } S.p > S.mmp \\ \text{take short position} & \text{if } S.p \leq S.mmp \end{cases} \quad (1)$$

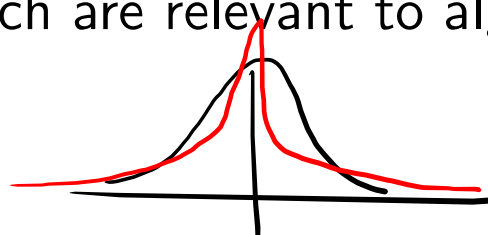
Reinforcement Learning (part of Machine Learning) is a convenient theoretical framework to develop trading strategies.

From Random Walk to Microstructure Data Series



Actual high-frequency financial time-series which are relevant to algorithmic trading exhibit the following properties

- Means very close to zero
- Extreme dispersion
- Dependence between successive observations
- Bid/Ask spread



~~i.i.d.~~

- Structural mean reversion component of traded prices.

Roll (1984) was one of the first scholars to propose a model of traded prices to take into account microstructure effects.

← permanent
transitory

The Roll Model of Traded Price - Introduction

Quoting Roll's paper abstract: In an efficient market, the fundamental value of a security fluctuates randomly. However, trading costs induce negative serial dependence in successive observed market price changes. This model is about the existence of an effective spread and the departure of traded prices from simple Brownian-like processes. References:

- A simple implicit measure of the effective bid–ask spread in an efficient market (Roll, 1984).
- Empirical Market Microstructure (Hasbrouck, 2007, Ch. 3).

The Roll Model of Traded Price - Definition

$$\begin{array}{c} \sigma_u^2 \\ c \end{array}$$

$$\begin{array}{c} m_t + c \\ m_t - c \end{array}$$

$$\rightarrow \quad m_t = m_{t-1} + \mu + u_t \quad (2)$$

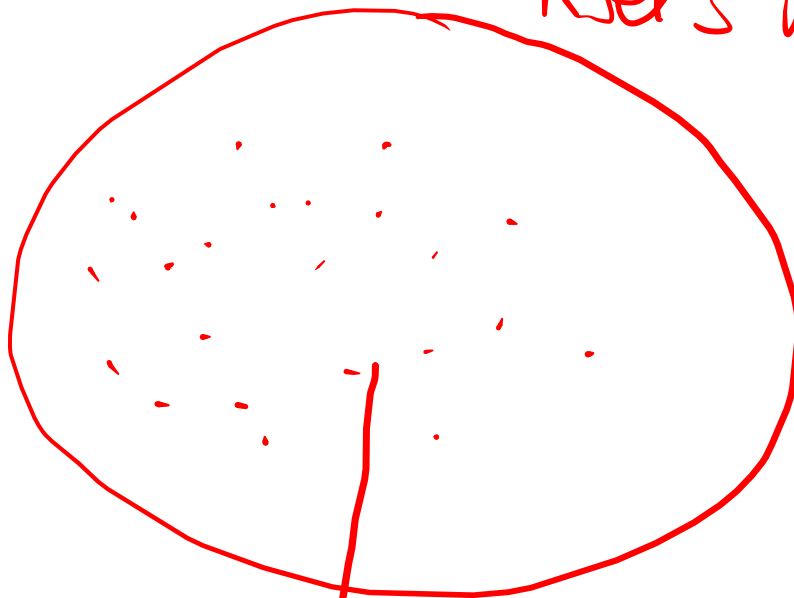
$$\rightarrow \quad p_t = m_t + cq_t \quad (3)$$

↑ perm. ← trans

where m_t represents the fundamental price (hidden), μ is the drift, usually discarded because negligible compared to u_t , u_t is a i.i.d. RV with zero mean and variance σ_u^2 , c represents a fixed cost incurred by liquidity providers to cover risk and fixed expenses, while $q_t \in \{-1, +1\}$ is also an i.i.d. RV with $\text{prob}(q_t = 1) = \text{prob}(q_t = -1) = 0.5$.

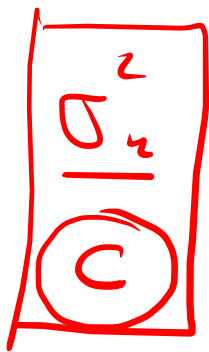
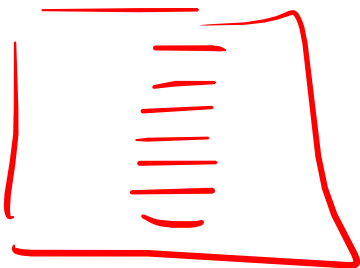
Moreover, here we assume that $E(u_t q_s) = 0$.

Rep's model



Data set

$\{P_t\}$



The Roll Model of Traded Prices - Calibration

$$\Delta p_t = p_t - p_{t-1} = m_t + c q_t - (m_{t-1} + c q_{t-1})$$

$$= \underline{u_t} + c \underline{q_t} - c \underline{q_{t-1}} = 0$$

If $\mu = 0$, $\Delta p_t \doteq p_t - p_{t-1}$, $\underline{E(\Delta p_t) = 0}$ then,

$$\gamma_0 \doteq \text{Var}(\Delta p_t) = E((p_t - p_{t-1})^2) = \sigma_u^2 + 2c^2 \quad (4)$$

$$\gamma_1 \doteq \text{Cov}(\Delta p_t, \Delta p_{t-1}) = E(\Delta p_t \Delta p_{t-1}) = -c^2 \quad (5)$$

Solving for the model's parameter,

$$\left[\begin{array}{l} c \\ \sigma_u^2 \end{array} \right] = \left[\begin{array}{l} \sqrt{-\gamma_1} \\ \gamma_0 + 2\gamma_1 \end{array} \right] \quad (6)$$

$$(7)$$

$$\begin{aligned} \text{Var}(\Delta p_t) &= E \left((\Delta p - \underbrace{E(\Delta p)}_0)^2 \right) \\ &= E(\Delta p_t^2) = \end{aligned}$$

$$= E \left((u_t + c q_t - c q_{t-1}) (u_t + c q_t - c q_{t-1}) \right) =$$

$$= E \left(u_t^2 + c^2 q_t^2 + c^2 q_{t-1}^2 \right)$$

$$\begin{array}{c} \downarrow \qquad \qquad \downarrow \\ \sigma_u^2 + c^2 + c^2 = \underbrace{\sigma_u^2 + 2c^2}_{= 80} \end{array}$$

Market Microstructure - Limit Orders and Central Order Books

Main parameters of a limit order are:

- Security and Market
- Buy/Sell
- Quantity
- Limit Price
- Time Validity

where the *price* parameter set the maximum a trader is willing to pay for a buy order, or the minimum is willing to get for a sell order.

A Central Order Book (COB) is a register maintained by the exchange where all incoming orders for a given security are recorded and matched if certain conditions are realized.

An open (not yet executed) order in the COB can be modified and/or canceled by a market participant, in the continuous phase.

BUY

SELL

Q	price	price	Q
200	<u>5.21</u>	5.24	300 ←
		5.25	400 100
		5.28	500

BUY ~~400~~ 5.25
300

Traded 200 5.24
100 5.24
300 5.25

Market Microstructure : Snapshot of an order book in a double auction market (best level only)

Time	Bid Qty	Bid Price	Ask Price	Ask Qty
15:40:00	15	24725	24730	8

Mini future on FTSE MIB equity index.

Best Ask Price: 24730. Quantity available to buyers: 8 lots.

Best Bid Price: 24725. Quantity available to seller : 15 lots.

Available Price for an immediate purchase up to 8 lots: 24730.

Available Price for an immediate sale up to 15 lots: 24725.

Mid Price: 24727.5

Other examples (Borsa Italiana, Yahoo, ..)